Title	Stars Are Not Pointless			
Creator:	Reger, Neil <u>nreger@access.k12.wv.us</u>	Reger, Neil <u>nreger@access.k12.wv.us</u>		
Source:	2009 TLI Staff PBL Plans			
Project Idea:	n-point, 3-dimensional stars are more purchasing several stars from the n department of your company found graphics art department concluded the stars would increase the aesthe potential customers would be asked 3-dimensional star. Prepare a prese includes detailed drawings, derivati different stars. Your drawings need horizontal distance (on the wall) fro radius), the measure of the interior tip of each point (on the wall), the h measures of the angles of each tria also develop formulas to find the ar minimize cost, each triangular secti sheet of material and assembled in	rea of each triangular face and the to ion of the star will be a separate pie the factory. Your presentation shou at with the least area needed to cons	stars on the market. After shape and design, the research t in the shape of their stars. Your rtion into the shape and design of he competitor's stars. Your of points they desire in their hitect Designs & Innovations that tifications of formulas for two o, the radius of the star, the x of the interior angles (interior r, the measure of the angle at the nce from the wall at the center), the s of each triangular face. You should otal area of the triangular faces. To ce punched from a rectangular ild include the procedure to find the	
Entry Event: Content Standards &	interest by sharing his knowledge a Provide the representative with the Discuss the Golden Ratio in nature	following ideas to guide their prese art, decorations and architectural de found all over the nation.	ecorations and architectural design. ntation.	
<b>Objectives:</b>	<b>M.O.T.3.8</b> investigate real-world problems within a project based investigation involving triangles using the trigonometric functions, the law of sines and the law of cosines, justify and present results.	Know the law of sines. Know the law of cosines. Know the definitions that relate the trigonometric functions to the sides of a right triangle Recognize a real-world situation that can be modeled by a triangle. Choose the appropriate method to solve a given triangle. Find all the missing parts of a triangle. Make a drawing to represent the	TargetActivity sheets from "Discovering Formulas to Solve Oblique Triangles"Notes from "Investigating Formulas to Solve and Find the Area of Oblique Triangles"Activity sheet from "Explore Oblique Triangles for the SAS case"Response sheet from "Explore Oblique Triangles for the SSA case"2-D Silhouette Example	

igonometry ge 2 of 11	[] <del>[</del>		
	M.O.T.3.9 develop and test a hypothesis to fi the area of a triangle given the measures of two sides and the included angle or the measures of three sides (Heron??s formula) and use these formulas to find total are of figures constructed of multiple shapes.	d triangle given SAS, AAS or ASA. Determine the appropriate formula. Determine how to separate a figure	Triangles for the SAS case"
21st Century Skills	Learning Skills & Technology Tools	Teaching Strategies Culminating Activity	Evidence of Success
Information and Communication Skills:	recognizes information needed for problem solving, can efficiently browse, search and navigate online to access relevant information, evaluates information based on credibility, social, economic, political and/or ethical issues, and presents findings clearly and persuasively using a range of technology tools and media.	The teacher will provide students the opportunity to make daily journal entries.	Students search and navigate the Internet to find relevant information related to Law of Sines, Law of Cosines, as well as special triangle area formulas. Students use graphing utilities and drawing software to separate a figure into multiple shapes. Students analyze various methods to solve triangles by using the given parts and determine the appropriate formula. Students communicate their problem solving methods using presentation software. Daily Writing Journal that includes accomplishments and reflections of lessons learned
	shares this information through a variety of oral, written and multimedia communications that target academic, professional and technical audiences and purposes. 21C.O.9-12.1.TT5 - Student uses advanced features of word processing software (e.g., outline, table of contents, index feature draw tool headers and	The teacher will monitor and assist students as they prepare and communicate their findings.	2-D Silhouette Example 3-D Example Demonstrating and Applying the Derived Formulas

age 3 of 11			
		The teacher will monitor and assist students as they use advanced features of word processing software in their products.	<u>Project Scenario</u>
Thinking and Reasoning Skills:	<ul> <li>21C.O.9-12.2.LS3 - Student engages in a problem solving process by formulating questions and applying complex strategies in order to independently solve problems.</li> <li>21C.O.9-12.2.TT3 - Student uses multiple electronic sources of information and multiple technology tools and resources tools (e.g., digital cameras, graphing calculators, probes, mp3 players, handheld devices, other emerging technologies, simulations, models, browsers, word processing, authoring tools, spreadsheets, databases) to collaborate with others, to formulate a hypothesis, to solve problems, make decisions, and</li> </ul>	The teacher will provide opportunities for students to separate real-world figures into multiple shapes with and without the use of technology. The teacher will monitor and assist students as they develop problem solving strategies. The teacher will monitor and assist students as use a variety of technology tools to solve problems.	Students use manipulatives to model separating a figure into multiple shapes. Students recognize ways to model real-world figures as a composite of multiple triangles. <u>3-D Example</u> <u>Demonstrating and Applying the</u> <u>Derived Formulas</u> <u>Project Scenario</u>
	present and justify the solutions. 21C.O.9-12.2.TT4 - Student uses technology tools and multiple media sources to analyze a real-world problem, design and implement a process to assess the information, and chart and evaluate progress toward the solution.	The teacher will monitor and assist students as use a variety of technology tools to analyze problems.	
Personal and Workplace Skills:	21C.O.9-12.3.LS2 - Student independently considers multiple perspectives and can represent a problem in more than one way, quickly and calmly changes focus and goals as the situation requires, and actively seeks innovations (e.g. technology) that will enhance his/her work.	Teacher presents real-world situations and monitors students in collaborative groups as they acquire and evaluate data in a variety of ways. The teacher will monitor and assist students as they work independently to create their products.	Students work in collaborative groups, choose appropriate tools, identify accurate models and evaluate conclusions in problem solving situations. 2-D Silhouette Example 3-D Example
	21C.O.9-12.3.LS5 - Student exhibits positive leadership through interpersonal and problem-solving skills that	The teacher will assist students self-monitor their group progress.	<u>Project Scenario</u> <u>Demonstrating and Applying the</u> <u>Derived Formulas</u>

Trigonometry Page 4 of 11		
	responsibilities effectively, and monitors group progress toward the goal without undermining the efforts of others. 21C.O.9-12.3.TT2 - Student the works collaboratively to acquire	he teacher will monitor and assist udents as they collaborative on the opropriateness of their information. he teacher will monitor and assist udents in their use of technology as ney identify and search needed iformation.
Performance Objectives:	Know Heron's formula Know the formulas to find area of <b>Do</b> Find all the missing parts of a triar Make a drawing to represent a sp Use the appropriate formula to find Use the appropriate formula to find Use multiple perspectives and can rep Exhibit positive leadership through in Work collaboratively to acquire inf Engage in a problem solving proce independently solve problems Use multiple electronic sources of Make informed choices among availa Use technology to seek strategies Use advanced features of word pr	aw of Cosines e trigonometric functions to the sides of a right triangle a triangle given SAS, AAS or ASA ngle ecific example d the area of a triangle and the area of figures edia sources to analyze a real-world problem present a problem in more than one way neterpersonal and problem-solving skills that contribute to achieving the goal formation from electronic resources ess by formulating questions and applying complex strategies in order to f information and multiple technology tools able advanced technology systems, resources and services a and information to address limits in knowledge rocessing and spreadsheet software kills of analysis, synthesis and evaluation and shares this information through a
Driving Question:		determine the design requirements of an n-point, 3-dimensional star?
Assessment Plan:	construct a 2-D silhouette example (o 3-dimensional star, to be used as mod choose the radius and the number of p members should choose a variety of r	an of your engineering division is asking each member of your team to outline of the star by tracing its borders on the wall) of an n-point, dels to check your team?s final formulas. Each member of your team needs to points for their star. To check the integrity of your final formulas, your team radii and number of points for their models. Create a spreadsheet that contains to check your final formulas. Your team must prepare a presentation to your

engineering division that details how each member constructed their star and determined the data values in the spreadsheet.Provide sound mathematical evidence for each detail.

3-D Example: The foreman of your engineering division is asking each member of your team to construct a 3-D example of an n-point, 3-dimensional star that can be placed on their 2-D silhouette example. This model will also be used to check your team?s final formulas. Each member of your team will use the radius and the number of points from their 2-D silhouette example. Add information that will be needed to check your final formulas to your spreadsheet from your 2-D silhouette example. Your team must prepare a presentation to your engineering division that details how each member constructed their star and determined the data values in the spreadsheet. Provide sound mathematical evidence for each detail.

**Demonstrating and Applying the Derived Formulas:** In only a few days, your engineering team will make their presentation to the design branch of Architect Designs & Innovations, detailing the derivation of formulas, and graphical representations that models the rectangular sheet with least area and the dimensions of an n-point, 3-dimensional star, given the radius and the number of points. The Chief Engineer is requesting from each of you, a persuasive essay that shows the derivation of formulas, demonstrates the use of formulas, shows drawings of your 2-D silhouette example and 3-D example, includes a detailed spreadsheet, and justifies the measures of your 2-D and 3-D examples using the derived formulas Provide sound mathematical evidence for each detail.

Project Scenario: You are engineers working for Architect Designs & Innovations. A major competitor claims that their n-point, 3-dimensional stars are more aesthetically pleasing than other stars on the market. After purchasing several stars from the major competitor and analyzing their shape and design, the research department of your company found that the Golden Ratio is not present in the shape of their stars. Your graphics art department concluded that incorporating the Divine Proportion into the shape and design of the stars would increase the aesthetic beauty, thus surpassing that of the competitor's stars. Your potential customers would be asked to provide the radius and number of points they desire in their 3-dimensional star. Prepare a presentation to the design branch of Architect Designs & Innovations that includes detailed drawings, derivation of all formulas and numerical justifications of formulas for two different stars. Your drawings need to show the use of the Golden Ratio, the radius of the star, the horizontal distance (on the wall) from the center of the star to the vertex of the interior angles (interior radius), the measure of the interior angle between the points of the star, the measure of the angle at the tip of each point (on the wall), the height of the star at the center (distance from the wall at the center), the measures of the angles of each triangular face and the lengths of sides of each triangular face. You should also develop formulas to find the area of each triangular face and the total area of the triangular faces. To minimize cost, each triangular section of the star will be a separate piece punched from a rectangular sheet of material and assembled in the factory. Your presentation should include the procedure to find the dimensions of the rectangular sheet with the least area needed to construct the star. Provide sound mathematical evidence for each detail.

	Major Group Products	<ul> <li>2-D Silhouette Example: Multimedia presentation that requires creat an organized spreadsheet and justification of numerical values in a two-dimensional model.</li> <li>3-D Example: Multimedia presentation that requires updating an organized spreadsheet and justification of numerical values in a three-dimensional model.</li> <li>Culminating Assessment (Project Scenario): Multimedia presentation research summary that requires the use of trigonometry to derive and apply formulas from 2-D and 3-D drawings and/or models.</li> </ul>		
	Major Individual Projects	<b>Demonstrating and Applying the Derived Formulas</b> : Persuasive essay that shows the derivation of formulas, demonstrates the use of formulas, shows 2-D and 3-D drawings and/or models, includes a detailed spreadsheet, and justifies the derived formulas to verify measurements in the 2-D and 3-D examples.		
Assessment and Reflection:	Rubric(s) I Will Use:	Collaboration	Written CommunicationXDemonstrating and Applyingthe Derived Formulas Rubric	

Map T

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		Critical Thinking & Pro Solving 2-D Silhouette Examp 3-D Example Rubric Demonstrating and Ap the Derived Formulas Project Scenario Rubr	nple Rubric <u>c</u> Applying las Rubric		Content Knowledge 2-D Silhouette Example Rubric 3-D Example Rubric Demonstrating and Applying the Derived Formulas Rubric Project Scenario Rubric		X
		Oral Communication 2-D Silhouette Examp 3-D Example Rubric Project Scenario Rubri		X	Other		
	Other Classroom Assessments For Learning:	Quizzes/ tests Teacher made quizzes/t	ests		Practice presentatio Practice Presentat		X
		Self-evaluation Architect Designs and Innovations Self-Tean Evaluation			Notes Individual student n	otes	X
		Peer evaluation Architect Designs and Innovations Self-Team Evaluation			Checklists/observat Architect Designs Innovations Adapt Knowledge Rating Vocabulary Develo Architect Designs Innovations Check	and_ ced Scale opment and	X
		Online Tests and Exa	ims		Concept Maps		Π
	Reflections:	Survey Architect Designs and Innovations Final Eva	-	X	Focus Group		
		Discussion Essential questions		.	Task Management ( Architect Designs : Innovations Check	and_	
		Journal Writing/ Learni Daily Writing Journal th includes accomplishme reflection of lessons lea	hat nts and a	L II.	Other Know-Need to Kno	ow Log	
The Product:	Product: Project Scenario						
	Knowledge and Skills	s Needed	Already Learn		e Taught Before the Project	Taught During the Project	,
	1. Recognize information needed for pl	roblem solving			X		
	2. Know the law of sines and cosines					X	
	3. Know the definitions that relate the trigonometric functions to the sides of a right triangle				X		
	4. Know Heron's formula					Х	
	5. Know the formulas to find area of a AAS or ASA	triangle given SAS,				x	
	6. Find all the missing parts of a triang	le				X	
	7. Make a drawing to represent a speci	fic example	X				
	8. Use the appropriate formula to find t and area of figures	the area of a triangle				х	

age 7 of 11						
	9. Use technology tools and multiple media sources to analyze a real-world problem		x			
	10. Use multiple perspectives and can represent a problem in more than one way	Х				
	11. Exhibit positive leadership through interpersonal and problem-solving skills that contribute to achieving the goal		х			
	12. Work collaboratively to acquire information from electronic resources			×		
	13. Engage in a problem solving process by formulating questions and applying complex strategies in order to independently solve problems		Х			
	14. Use multiple electronic sources of information and multiple technology tools			x		
	15. Make informed choices among available advanced technology systems, resources and services			x		
	16. Use technology to seek strategies and information to address limits in knowledge			x		
	17. Use advanced features of word processing and spreadsheet software			x		
	18. Create information using advanced skills of analysis, synthesis and evaluation and shares this information through a variety of oral, written and multimedia communications			х		
Resources:	School-based Individuals: Technology Integration Specialist					
	Technology:         Computer Lab         Internet Browser with Java enabled         Word Processing Software         Spreadsheet Software         Presentation Center         Resource/Learning Center:         Area Resources         Constructing Stars         Derivations and Proofs of Formulas         Golden Ratio Resources         Law of Cosines         Law of Sines         Oblique Triangle Resources         Pictures of Country Stars         Discovering Formulas to Solve Oblique Triangles:         http://illuminations.nctm.org/Lessons/LawSinesCosines/Law         http://illuminations.nctm.org/Lessons/LawCosinesGeo/LawC         Investigating Formulas to Solve and Find the Area of Oblique         http://www.algebralab.org/studyaids/studyaid.aspx?file=Trig         http://www.algebralab.org/studyaids/studyaid.aspx?file=Trig         http://www.sascurriculumpathways.com -> Mathematics -> To         Classroom Activity 1047         Explore Oblique Triangles for the SSA case:         http://www.sascurriculumpathways.com -> Mathematics -> To         Web Inquiry 116         Writing a Persuasive Essay:	<u>CosinesGeo-As</u> Triangles: <u>conometry_Lav</u> <u>conometry_Lav</u> Trigonometry -	<u>S-SquareTriangl</u> <u>vSines.xml</u> vCosines.xml > Triangles: Rig	ht & Oblique ->		

	Community: Representatives from Decorations and/or Architectural Design Businesses
	Materials: Rulers Protractors Compasses Graphing calculator Poster board or other construction material
Manage the	Project Storyboard: Allow approximately 5 weeks.
Process:	Before the project begins: Divide students into teams of 3 or 4 students for major group products and projects.
	Prepare a Resource/Learning Center for differentiating and tiering. Include the following possible tips or hints (mathematical knowledge students will need to know to complete this project) in the project Resource/Learning Center: <u>Area Resources</u> <u>Constructing Stars</u>
	Derivations and Proofs of Formulas Golden Ratio Resources
	Law of Cosines Law of Sines
	Oblique Triangle Resources
	Pictures of Country Stars
	As a homework assignment at the end of each day, each student will use a word processor to keep a daily writing journal that includes accomplishments and reflections of lessons learned. All entries will be in complete sentences.
	Launch the Project. Driving Question: How can trigonometry be used to determine the design requirements of an n-point, 3-dimensional star?
	<b>Entry Event</b> : Invite a representative from a local decorations and/or architectural design businesses to initiate student interest by sharing his knowledge about using the golden ratio in art, decorations and architectural design. Provide the representative with the following ideas to guide his presentation. Discuss the Golden Ratio in nature.
	Discuss using the Golden Ratio in art, decorations and architectural design.
	Look at <u>Golden Ratio Resources</u> . Discuss how Country Stars can be found all over the nation.
	Look at <u>Pictures of Country Stars</u> .
	Distribute the <b>Project Scenario</b> to each student.
	Distribute Know/Need to Know Log to individual teams to be used as periodic formative assessment.
	Distribute <u>Architect Designs and Innovations Team Roles</u> descriptions to each student. For groups of 4 students, two of the students can share the responsibilities of Design Engineer or Research Engineer. As an assignment, each team submits a Team Contract. Examples and ideas for writing contracts can be found at <u>http://www.google.com</u> and searching "employment agreement contracts."
	Distribute Architect Designs and Innovations Checklist to each team.
	Distribute <u>Architect Designs and Innovations Adapted Knowledge Rating Scale Vocabulary Development</u> to each student. Use as formative assessment. Students update the vocabulary development throughout the PBL experience.
	Students will use a word processor to begin a daily writing journal that reflects on a summary of the

lessons learned.

Discovering Formulas to Solve Oblique Triangles. Essential Question: How can geometry and right triangle trigonometry be used to solve oblique triangles?

Students will work in pairs or teams to investigate and explore the Law of Sines. Print <u>http://illuminations.nctm.org/Lessons/LawSinesCosines/LawSines-AS-Discover.pdf</u> (The Law of Sines) Activity Sheet. Each team completes the activity sheet. Each team will submit a copy of their activity sheet for evaluation.

Students will work in pairs or teams to investigate and explore squares on a triangle to discover the Law of Cosines. Print

<u>http://illuminations.nctm.org/Lessons/LawCosinesGeo/LawCosinesGeo-AS-SquareTriangle.pdf</u> (Squares on a Triangle) Activity Sheet. Each team shares a workstation and uses <u>http://illuminations.nctm.org</u> to complete the activity sheet. Each team will submit a copy of their activity sheet for evaluation.

<u>2-D Silhouette Example</u> Challenge.
Essential Question: How can a 2-dimensional model help to understand 3-dimensional space?

Distribute 2-D Silhouette Example challenge to each team.

Investigating Formulas to Solve and Find the Area of Oblique Triangles. Essential Question: How can solving and finding the area of oblique triangles be useful in the real world?

Students will use a word processor and work in pairs or teams to take notes from <a href="http://www.algebralab.org/studyaids/studyaid.aspx?file=Trigonometry\_LawSines.xml">http://www.algebralab.org/studyaids/studyaid.aspx?file=Trigonometry\_LawSines.xml</a> on the Law of Sines and the area of Oblique Triangles. Each team will submit a copy of their notes for evaluation.

Students will use a word processor and work in pairs or teams to take notes from <a href="http://www.algebralab.org/studyaids/studyaid.aspx?file=Trigonometry\_LawCosines.xml">http://www.algebralab.org/studyaids/studyaid.aspx?file=Trigonometry\_LawCosines.xml</a> on the Law of Cosines and the area of Oblique Triangles. Each team will submit a copy of their notes for evaluation.

**<u>3-D Example</u>** Challenge.

Essential Question: Why is the use of a scale model important in mathematics?

Distribute <u>3-D Example</u> challenge to each team.

Presentation of <u>2-D Silhouette Example</u>. (This presentation can be used as a practice presentation. For the practice presentation, use <u>Practice Presentation Checklist</u>.)

Explore Oblique Triangles for the SAS case. Essential Question: How can the right triangle, area formula be used to derive other area formulas?

Students will work in teams to investigate and explore oblique triangles. Students will use their notes from

"Investigating Formulas to Solve and Find the Area of Oblique Triangles." Teams use <u>http://www.sascurriculumpathways.com</u> -> Mathematics -> Trigonometry -> Triangles: Right & Oblique -> Classroom Activity 1047 to explore oblique triangles, in particular the case in which two sides and the included angle (SAS) are given. Finding a More General Formula Activity Sheet can be used as formative assessment.

Explore Oblique Triangles for the SSA case. Essential Question: Does every triangle have a unique solution?

Students will work in teams to investigate and explore oblique triangles. Students will use their notes from

"Investigating Formulas to Solve and Find the Area of Oblique Triangles." Teams use <u>http://www.sascurriculumpathways.com</u> -> Mathematics -> Trigonometry -> Triangles: Right & Oblique -> Web Inquiry 116 to explore oblique triangles, in particular the case in which two sides and the non-included angle (SSA) are given. The Response Sheet can be used as formative assessment.

	Demonstrating and Applying the Derived Formulas Challenge. Essential Question: Why is the use of models important in mathematics?
	Distribute <b>Demonstrating and Applying the Derived Formulas</b> challenge to each team member.
	For help with elements of a persuasive essay, students should visit <u>http://www.studygs.net/wrtstr4.htm</u> .
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## Teach21 Teach21 Project Based Learning Stars Are Not Pointless Trigonometry Page 11 of 11

• UP3370WS15.doc (http://wveis.k12.wv.us/Teach21/CS0/Upload/UP3370WS15.doc)
• UP3370WS16.doc (http://wveis.k12.wv.us/Teach21/CS0/Upload/UP3370WS16.doc)
• UP3370WS17.doc (http://wveis.k12.wv.us/Teach21/CS0/Upload/UP3370WS17.doc)
• UP3370WS18.doc (http://wveis.k12.wv.us/Teach21/CSO/Upload/UP3370WS18.doc)
• UP3370WS19.doc (http://wveis.k12.wv.us/Teach21/CSO/Upload/UP3370WS19.doc)
• UP3370WS20.doc (http://wveis.k12.wv.us/Teach21/CS0/Upload/UP3370WS20.doc)
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